# A 13Ghz Loadable Counter with 20ps/bit Settling Time and Early Completion, in 40nm CMOS 

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## Outline

- Numbers
>How to count
>Problem statement
>Number representations
>Redundant representations
>Resettling
- Circuits
>Algorithm
$>$ Circuit
>Layout
$>$ Implementation
>Demo
- Interlude


## How do we count?

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- Counting down


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$>$
17


## How do we count?

- Counting down
$>$
17
$>$
16


## How do we count?

- Counting down

| $>$ | 17 |
| :--- | :--- |
| $>$ | 16 |
| $>$ | 15 |

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| $>$ | 17 |
| :--- | :--- |
| $>$ | 16 |
| $>$ | 15 |
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| $>$ | 16 |
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| $>$ | 12 |

## How do we count?

- Counting down

|  | 17 |
| ---: | ---: |
| $>$ | 16 |
| $>$ | 15 |
| $>$ | 14 |
| $>$ | 13 |
| $>$ | 12 |
| $>$ | ... and so on |

## How do we count?

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- Counting down in binary


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$>10001=17$


## How do we count?

- Counting down in binary

$$
\begin{aligned}
& >10001=17 \\
& >10000=16
\end{aligned}
$$

## How do we count?

- Counting down in binary
$>10001=17$
$>10000=16$
$>01111=15$


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$>10001=17$
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$>10001=17$
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$>01100=12$


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- Counting down in binary
$>10001=17$
$>10000=16$
$>01111=15$
$>01110=14$
$>01101=13$
$>01100=12$
$>\ldots$ and so on


## How do we count?

- Counting down in binary
$>10001=17$ Problem: "zeroness" may depend
$>10000 \leftrightarrows 16$
$>01111=15$
$>01110=14$
$>01101=13$
$>01100=12$
$>\ldots$ and so on


## How do we count?

5. A shovel that normally costs $\$ 8$ is on sale for $40 \%$ off. What is the sale price of the shovel?


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## Even my students understand this

## What problem are we solving?

- Need an n-bit counter with two operations:
> load will:
$>$ Accept an ordinary binary value (no fancy encodings allowed!)
$>$ Set the counter to that value
> dec will either:
$>$ Report failure if the counter value was zero
$>$ Report success and decrement the counter if it was nonzero
- Performance requirements:
>"dec" must complete in a bounded amount of time
$>$ No matter what the count value is.
$>$ No matter how big the counter $(\mathrm{n})$ is.


## How do we count?

- Counting down in binary
$>10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17$
$>10000=16$
$>01111=15$
$>01110=14$
$>01101=13$
$>01100=12$
$>\ldots$ and so on


## How do we count?

- Redundant binary representations

$$
>10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17
$$

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\begin{aligned}
& >10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >02001=0 \times 2^{4}+2 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17
\end{aligned}
$$

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- Redundant binary representations

$$
\begin{aligned}
& >10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >02001=0 \times 2^{4}+2 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >01201=0 \times 2^{4}+1 \times 2^{3}+2 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17
\end{aligned}
$$

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- Redundant binary representations

$$
\begin{aligned}
& >10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >02001=0 \times 2^{4}+2 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >01201=0 \times 2^{4}+1 \times 2^{3}+2 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
& >01121=0 \times 2^{4}+1 \times 2^{3}+1 \times 2^{2}+2 \times 2^{1}+1 \times 2^{0}=17
\end{aligned}
$$

## How do we count?

- Redundant binary representations

$$
\begin{gathered}
>10001=1 \times 2^{4}+0 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
>02001=0 \times 2^{4}+2 \times 2^{3}+0 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
>01201=0 \times 2^{4}+1 \times 2^{3}+2 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}=17 \\
>01121=0 \times 2^{4}+1 \times 2^{3}+1 \times 2^{2}+2 \times 2^{1}+1 \times 2^{0}=17 \\
\text { Fully "settled" representation }
\end{gathered}
$$

## How do we count?

- Decrementing a settled number

$$
\begin{aligned}
& >01121=0 \times 2^{4}+1 \times 2^{3}+1 \times 2^{2}+2 \times 2^{1}+1 \times 2^{0}=17 \\
& >01120=0 \times 2^{4}+1 \times 2^{3}+1 \times 2^{2}+2 \times 2^{1}+0 \times 2^{0}=16
\end{aligned}
$$

Theorem 1: if the representation is fully settled, only the least significant bit needs to be used to decrement and test zeroness.

## How do we count?

- Resettling
$>01120=16$
$>01112=16$


## How do we count?

- Resettling
$>01120=16>01110=14$
$>01112=16>01102=14$


## How do we count?

- Resettling
$>01120=16 \quad>01110=14$
$>01112=16 \quad>01102=14$

Any time you see a 0 to the right of a non- 0 , you can resettle by decrementing the non-0 and turning the 0 into a 2

## How do we count?

- Theorem 2 (Ebergen)
>On average, no more than 2 settling operations are performed per decrement operation.

$$
\begin{aligned}
& >01121=17 \\
& >01120=16 \\
& >01112=16 \\
& >01111=15 \\
& >01110=14 \\
& >01102=14 \\
& >01022=14 \\
& >00222=14 \\
& >00221=13 \\
& >00220=12 \\
& >00212=12 \\
& >00211=11
\end{aligned}
$$

## How do we count?

- Theorem 2 (Ebergen)
>On average, no more than 2 settling operations are performed per decrement operation.

$$
\begin{aligned}
& \\
>01121 & =17 \\
>01120 & =16 \\
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>01102 & =14 \\
>01022 & =14 \\
>00222 & =14 \\
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>00220 & =12 \\
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>00211 & =11
\end{aligned}
$$

## How do we count?

- Theorem 2 (Ebergen)
>On average, no more than 2 settling operations are performed per decrement operation.

$$
\begin{aligned}
& \text { Toggle rate is " } n / 2 " \\
>01121 & =17 \\
>01120 & =16 \\
>01112 & =16 \\
>01111 & =15 \\
>01110 & =14 \\
>01102 & =14 \\
>01022 & =14 \\
>00222 & =14 \\
>00221 & =13 \\
>00220 & =12 \\
>00212 & =12 \\
>00211 & =11
\end{aligned}
$$

## How do we count?

- Theorem 2 (Ebergen)
>On average, no more than 2 settling operations are performed per decrement operation.

$$
\begin{aligned}
& > 0 1 \longdiv { 1 2 1 } = 1 7 \\
& >01120=16 \\
& >01112=16 \\
& >01111=15 \\
& >01110=14 \\
& >01102=14 \\
& >01022=14 \\
& >00022=14 \\
& >00221=13 \\
& >00220=12 \\
& >00012=12 \\
& >00211=11
\end{aligned}
$$

## How do we count?

- Theorem 2 (Ebergen)
>On average, no more than 2 settling operations are performed per decrement operation.

```
>0 1121 = 17 Toggle rate is " }n/8
>01121 = 17
>01120 = 16
> 91112 = 16
>01111 = 15
>01110 = 14
>01102 = 14
> 11022 = 14
>00222 = 14
>00221 = 13
>00220 = 12
>00212 = 12
>00211 = 11
```


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- Theorem 2 (Ebergen)
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\begin{aligned}
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& >00222=14 \\
& >00221=13 \\
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>01121 & =17 \\
>01120 & =16 \\
>01112 & =16 \\
>01111 & =15 \\
>01110 & =14 \\
>01102 & =14 \\
>01022 & =14 \\
>00222 & =14 \\
>00221 & =13 \\
>00220 & =12 \\
>00212 & =12 \\
>00211 & =11
\end{aligned} \quad \sum_{i=0}^{\infty} \frac{n}{2^{i}}=n / 1+n / 2+n / 4+\ldots=2 n
$$

## How do we count?

- Resettling

$$
\begin{aligned}
& >01112=16 \\
& >01111=15 \\
& >01110=14 \\
& >01102=14 \\
& >01021=13
\end{aligned}
$$

Theorem 3: resettling the upper bits can be performed concurrently with decrementing the lower bits. If the number was already settled, it will resettle as fast or faster than it can be decremented.

## Interlude

- Have we assumed that the counter is only finitely long?


## Circuits

## Syntax

- To avoid terrible confusion, I will...
$>$ Use words for states: Zero, One, Two, Done
$>$ Use numbers for numerals (duh): 0, 1, 2
$>$ Use symbols for logic levels: -, +


## Circuit Implementation

- Each "bit" of the counter is a pair of state wires
$>$ Four possible states: Zero, One, Two, Done
- A GasP module sits between each pair of bits
$>$ When the more significant neighbor is not Zero
>.. and the least significant neighbor is Zero
$>$ then fire, and:
>If the more significant neighbor is Done set the less significant neighbor to Done
>Otherwise decrement the more significant neighbor and set the less significant neighbor to Two


## Safest Binary Encoding

- I will name the two state wires
>OneOrDone, which is + when the state is One or Done
> OneOrTwo, which is + when the state is One or Two
- I will write the state of a pair of wires as
> [OneOrDone, OneOrTwo]
$>$ So,
$>$ Zero $=[--]$
$>$ One $=[++]$
$>$ Two $=[-+]$
$>$ Done $=[+-]$


## Hamming Distances

- Diagram
>Hamming-adjacent codes connected by dashed lines
>Arrowheads indicate possible transitions
- One transition (One-toZero) is not Hamming adjacent
>Neighbor might "see" a Done or a Two during the transition.
>Must manually check that this
 will not cause misbehavior.
$>$ Fortunately, it does not.


## Adding a Timing Constraint

- If we are willing to assume a timing constraint, we can simplify the circuit
$>$ This turns out to speed it up considerably
$>$ New state wires:
>TwoOrDone, which is + when the state is Two or Done
>TwoOrOne, which is + when the state is Two or One


## Half-Drivers

- Pull-up network:
> active only when output is high
$>$ weak ( $\mathrm{X}=1$ ) transistors
>otherwise same behavior



## One Bit (Full Circuit)



## Loading 16 (binary 10000)



## Loading 18 (binary 10010)

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## 40nm Implementation

- Loadable down counter designed and implemented
- Summary
>Calibre DRC clean
>Simulation from extracted layout
>486 lambda bit pitch (x 3 rows tall)
>Ready for tape-out (just needs fill+scan)
>Performance: 13Ghz (76ps cycle), 20ps/bit settling time


## 40nm Layout

## - Calibre DRC clean - 486 $\lambda$ x $810 \lambda$ per bit - M1+M2 only



## Marina test chip

- Includes earlier 6/4 GasP counter design, 90nm
$>6$ bits wide
$>$ Fully interfaced to Dock



## Marina Demo

- Caveats
$>$ This design was done before the 40 nm counter I just presented
$>$ This design was done in three weeks, conception to tape-out
$>$... because we finished the main project early and had extra space
$>$ This design is in 90 nm CMOS, not 40 nm
$>$ The counter is deployed in an application
$>$ No test harness, so measuring performance is a bit tricky


## Marina Demo

- Program you will see (for varying values of $X$ ):
>Repeat forever:
$>$ Load counter with $X$
$>$ Run the counter down
$>$ Send a token
- Token pulses are passed through 16 frequency dividers (each divides by two) before going to the pad.


## Demo

## The Power of Asynchrony

- Different bits need not be sized the same!
$>$ No clock constraint to meet, so:
$>$ Size the least significant bits very large (fast, lots of area)
>Size the more significant bits exponentially smaller
- Down to min-size
- Big area savings in large (>=64bit) counters



## What does this have to do with Fleet? (important slide)

- In a conventional processor, the clock is the "animating force" which drives computation forward
- In Fleet, the "animating force" is a counter running down >Every asynchronous system is "just" a mass of coupled ring oscillators
$>$ Voltages move around rings like teeth of gears
>Counters are the engines which drive the network of gears
- Fast, wide counters are important for fast Fleets


## Questions?

